



Professionally Managed, Publicly Traded Commodity Funds

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Professionally Managed, Publicly Traded Commodity Funds*

The best laid schemes o' mice and men
Gang aft a-gley;
An' lea's us nought but grief an' pain,
for promis'd joy.

[Robert Burns]

The concept of a professionally managed investment portfolio composed of positions in commodity and financial futures is appealing to the investment community. One thinks of investment in commodities as a way to protect capital in times of high inflation. Perhaps this partially accounts for the tremendous growth of professionally managed public commodity funds. Prior to February 1979 there was only one fund for which monthly data existed, but by 1985 the number had grown to 94 funds, with over \$600 million in assets under management.¹

Most of the funds have the ability to trade in all available futures and commodities. These include financial instruments and futures on

Investment in professionally managed, publicly traded commodity funds has grown rapidly in recent years.

This is the first comprehensive study of the performance of these funds. It is found that randomly selected funds offer neither an attractive alternative to bonds and stocks nor a profitable addition to a portfolio of stocks and bonds. Furthermore, past performance of these funds offers very little information about future performance.

The findings may be explained by the large transaction costs incurred by these funds and their primary reliance on technical analysis.

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1. *Managed Accounts Report* (1979-85). We would like to thank Morton Baratz for helpful comments and information.

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financial instruments, foreign currencies and futures and forward contracts on foreign currencies, and commodities and futures on commodities. Most funds do not restrict themselves to subsectors of the market but take positions across all sectors; funds can and do take both long and short positions in the futures market. A typical quote from a prospectus is as follows: "Diversification will be achieved by trading a variety of commodities. . . ."²

Most commodity funds trade on the basis of technical systems. In particular, trend-following systems that assume that a trend in price will continue over time are typical. Here is a representative statement from a prospectus: "Trading decisions of the Advisor will be based primarily on technical analysis and trend-following trading strategies which seek to identify price changes and trends. The buy and sell decisions based on these strategies are not based on analysis of fundamental supply and demand factors, general economic factors or anticipated world events."³

Given the popularity of these funds and their probable continued growth, it seems worthwhile to examine their performance. Interest in their performance is increased because of the apparent handicaps that investors in these funds face. These handicaps are of two types. First, their management fees and transaction costs are very high compared with transaction costs and management fees for more traditional fixed-income or equity investments. Yearly management fees and transaction costs of commodity funds have been estimated to average over 19.2% of assets under management.⁴ Second, most commodity fund managers appear to rely primarily on trend following and technical trading rules for decision making. These rules have been discredited in the common stock area. In fact, many of these rules were first tested (and found not to work) on commodity data (e.g., Mandelbrot 1966).

In this paper we will examine the performance of publicly traded commodity funds. One problem encountered in the analysis is that the data sample is small since these funds have not been in existence very long. There are only 6 years of performance figures. In addition, as we will see, the standard deviation of return on these funds is extremely large relative to other investments. This makes the problem of analysis from 6 years of data even more difficult.

This paper is divided into four sections. In Section I we discuss the data we analyze in the subsequent sections. In Section II we examine the performance of publicly traded commodity funds. In particular, we

2. Prospectus of Dean Witter Reynolds Commodity Partners (April 1985), p. 33.

3. Prospectus of Matterhorn Commodity Partners (April 9, 1981), p. 11.

4. See Elton and Gruber (1987, ch. 20) for a discussion of transaction costs on futures and commodity funds, and see Irwin and Brorsen (1985).

consider whether it is optimal either to hold such a fund in isolation or to add it to an existing portfolio of stocks and bonds. In Section III we examine whether we can differentiate between funds that will do well and funds that will do poorly in the future. If past performance data are indicative of future performance in either an absolute or a relative sense, then performance can be improved beyond that found in Section II. Section IV contains our conclusion.

I. Data

The basic data set employed in this paper is the monthly total rate of return on each publicly traded commodity fund in existence from July 1979 to June 1985. Total return is defined as the cash distribution during any month plus the change in net asset value over the month divided by the value per unit at the beginning of the month.⁵ When we performed this study, the latest data available were for the month of June 1985. Thus, we ended our last year in June 1985. We then had 6 years' worth of usable data. We could have started the data series 1 year earlier, but there were monthly data on only one fund in that year. Table 1 shows the number of funds in existence in June of each sample year that were publicly reporting monthly data.⁶ When a fund dissolved midyear, we computed returns as if the proceeds were invested in the average commodity fund. However, dissolved funds were not included in standard deviation calculations.

In order to make some comparisons with alternative investments, we also employed indexes of monthly return performance on several other types of investments. Monthly returns on the Standard and Poor's 500 Stock Index, Treasury bills, long-term government bonds, long-term corporate bonds, and small stocks were taken from Ibbotson (1985). In addition, for our 6-year sample period the Shearson Bond Index was employed. Finally, to get some idea of how a long market position in commodities and futures might perform, monthly rates of return were calculated from the Dow Jones Spot Index and the Dow Jones Futures Index.

5. Generally, publicly traded commodity funds allow investors to liquidate their holdings on the last day of any month at their net asset value. Risk from these funds is in reality higher relative to other investments than the numbers in subsequent tables indicate because the alternative investments can be liquidated at any time during the month.

6. Eight funds dissolved due to poor performance during our sample period. One more dissolved during the period but was not included in our sample because it did not exist during any June. Three more went bankrupt shortly after our period. Dissolution values were not obtainable for three funds. For these funds we assumed dissolutions at their prior months' asset value. This overstates returns.

TABLE 1 Return and Risk Data for Commodity Funds

Year	No. of Funds	Average Monthly Return		Average SD
		For Monthly Holding Period	For Annual Holding Period	
1	12	.0182	.0027	.1577
2	16	.0219	.0090	.1211
3	34	.0149	.0112	.0824
4	49	-.0191	-.0267	.1167
5	70	-.0020	-.0054	.0793
6	85	.0097	.0048	.0943
Average	...	.0073	-.0007	.1130*

* Not simply an average of the six yearly numbers. Rather, it is an average arrived at using the technique discussed in the text.

II. The Performance of Publicly Traded Commodity Funds

In this section, we will look at the desirability of holding a randomly selected, publicly traded commodity fund. The appropriate way to analyze this question is to examine whether such a commodity fund should be added to a bond or stock portfolio. However, before doing so, we will briefly examine whether a randomly selected commodity fund is a desirable stand-alone investment.

A. Commodity Funds as an Alternative Investment Vehicle

Assume an investor, at the beginning of each year, selected a commodity fund at random. What would the performance look like? Table 1 gives a partial answer. Table 1 lists the average monthly returns (assuming both a 1-month and a 1-year holding period), the average standard deviation of returns, and the number of funds in our study at the beginning of each year. To provide points of comparison, table 2 shows some data for stocks and bonds. All of this data was also computed using a 1-year holding period.

The data shown for common stocks, small stocks, and the Shearson Bond Index can be considered typical of the performance of a widely diversified passive investment in each type of security. The common stock data are for the Standard and Poor's 500 Index and would be reflective of the results for most index funds before management fees. The small stock data are for the 20% of the stocks with the lowest capitalization on the NYSE. The amount invested in each stock is directly proportional to the stock's capitalization. Finally, the Shearson index is an index of all government and corporate bonds. Like the stock indexes described above, it is a capitalization-weighted index with the weights directly proportional to the bond's capitalization. Government bonds constitute about 75% of the Shearson Bond Index, and the index is dominated by bonds of intermediate maturity. The

TABLE 2 Return and Risk Data for Comparable Assets

	June 1960–June 1985		Sample 6-Year Period	
	Average Monthly Returns (Annual Holding Period)	SD Monthly Returns*	Average Monthly Returns (Annual Holding Period)	SD Monthly Returns
Common stocks	.0074	.0414	.0131	.0399
Small stocks	.0123	.0639	.0168	.0465
Long-term corporate bonds	.0045	.0271	.0079	.0428
Long-term govern- ment bonds	.0041	.0279	.0075	.0435
Shearson Bond Index	N.A.	N.A.	.0097	.0293
Treasury bills	.0051	.0026	.0085	.0015

SOURCE.—Ibbotson (1984), except for Shearson Bond Index.

* 25 years ending December 1984.

Shearson Bond Index is not used over the entire 25-year period because it has only been computed for recent years.

The Ibbotson and Sinquefeld indexes (Ibbotson 1985) are indexes for single 20-year bonds. These indexes, which have become a standard for the industry, are employed here because, unlike the Shearson Bond Index, they exist over a long time period. The disadvantage of the Ibbotson and Sinquefeld data for our purpose is that they may not be an accurate representation of a typical bond portfolio. The typical bond portfolio contains many bonds and has a shorter duration. The latter will surely cause differences in return. We would expect a bond portfolio to have less risk than the Ibbotson and Sinquefeld data both because of the diversification effect of multiple bonds and because of shorter duration. This is reflected in our sample period. The standard deviation of the Shearson Bond Index is .0293 compared with .0435 for the Ibbotson and Sinquefeld government bond index over the same time period.

Before examining commodity funds as an investment vehicle, one other issue needs to be discussed. We are using a mean-variance framework. On the surface it might seem that commodity funds could serve as a hedge against shifts in the prices of consumption goods or inflation. If this were true, then a more general framework such as Long's (1974) or Breeden's (1979) equilibrium model might be necessary to examine their relative value. However, a little reflection shows that this is an inaccurate perspective. As discussed before, commodity funds do not specialize in a commodity or group of commodities. They can hold commodities, commodity futures, financial instruments, financial futures, currencies, and currency futures long or short. Some

months a fund can be long a commodity position, and the next period it may be short the same commodity position. Funds allow all these possibilities in their prospectuses. Thus an investor could not analyze a prospectus and determine that a particular commodity fund was a potential hedge against a price rise in any commodity or inflation in general. Further, as we discuss later, the actual return data does not allow an investor to determine which funds might serve as a hedge against inflation. The rank correlation between funds from one period to the next ranked on correlation with inflation is insignificant and sometimes positive and sometimes negative. The correlations of the average fund with inflation is close to zero. Thus an investor could not use an existing commodity fund as a potential inflation hedge, and therefore a general mean-variance framework is appropriate.

The most striking characteristic of commodity funds relative to bonds and stocks is their high standard deviation. The standard deviation of the typical commodity fund is more than three times as high as that of the bond index and more than twice as high as the stock index for most years.

There are two different monthly returns displayed in table 1. The monthly return labeled "monthly holding period" literally applies only if an investor has a 1-month holding period. The average monthly return with an annual holding period is the twelfth root of the annual return minus one. Why the difference in the two numbers? To get an annual return, one compounds the monthly returns. The average monthly return for the 1-year holding period is the twelfth root of this annual rate. This is, of course, a geometric average, while the figures shown in table 1 under the heading "monthly holding period return" are an arithmetic average. To see what a difference a change in holding period can make, let us examine one of the 12 firms in year 1. It had an arithmetic average return of 9.6% per month, or a 200% annualized return. The actual yearly return was 35%, or 2.53% per month. One reason for this difference was a decline in 1 month of 43% followed by a rise of 85%. The arithmetic mean of this is 21% while the geometric mean is 2.7%. We feel the assumption of an annual holding period is likely to be more relevant. Few, if any, investors have an investment horizon of only 1 month. In addition, the sales fees (for purchase of commodity funds) would virtually ensure a negative return on a 1-month holding period basis.

The same kind of considerations hold, of course, for bonds and stocks. We calculated these returns on the same annual holding period basis.⁷

How do publicly traded commodity funds compare as an investment

7. Actually, the difference in return for these instruments is so small that it makes much less difference whether a 1-month or 1-year holding period is assumed.

alternative to bonds or stocks? As discussed earlier, the risk for commodity funds is two to three times larger than for stock or bond funds. While 6 years is a short period to generalize from, an examination of table 1 shows that returns on commodity funds were poor relative to actual performance on stocks and bonds over the sample period and relative to our expectations concerning stock and bond returns in the long run. The average return on commodity funds was negative in 2 of the 6 years and negative overall.⁸ For the 266 fund years in our sample, 130 were negative and 136 were positive. These numbers assume that each fund year is equally likely to be picked. Since we have more funds in later years, it emphasizes performance in these years. If we assume that each year is equally likely to be picked and then randomly select a fund within that year, then the probability of picking a fund with a positive return is .56. Commodity funds use Treasury bills as collateral for their futures contracts. With zero selection ability and no management fees or transaction costs we should observe a return equal to that on Treasury bills. We found that, of the 266 fund years in our sample, 162 funds had returns below this benchmark and 104 had returns above. The average probability over the 6 years of selecting a fund at random with a return above the riskless rate was .42. Over our sample period, the probability that the mean return on commodity funds was equal to or greater than the mean return of either the stock fund or the bond fund was about 5%.⁹ Thus both risk and return considerations would suggest that commodity funds are not a useful stand-alone investment.

While commodity funds do not seem to be an attractive alternative to a portfolio of bonds or stocks, it is still possible that they are an appropriate *addition* to a portfolio of bonds and/or stocks. This depends not only on the expected value and standard deviation of fund returns but also on how these returns are correlated with bond and stock returns. It is to this issue that we now turn.

B. Commodity Funds as Part of an Overall Portfolio

In this section we will examine whether a commodity fund selected at random should be added to a portfolio of stocks and bonds. In Appendix A, we show that a commodity fund should be added as long as¹⁰

$$\frac{\bar{R}_c - R_f}{\sigma_c} > \left(\frac{\bar{R}_p - R_f}{\sigma_p} \right) \rho_{cp}, \quad (1)$$

8. We computed both Sharpe and Jensen performance measures. For the Jensen performance measures, we used both an index of bonds and an index of stocks. All performance measures had negative values, indicating that commodity funds should not be held. Actually, this result could have been anticipated from the numbers presented in tables 1 and 2.

9. The standard deviation of mean returns across the 6 years is .0052.

10. An equivalent condition is presented in Blume (1984).

where $\bar{R}_c$ is the expected return of commodity fund c , R_f is the riskless rate, σ_c is the standard deviation of the commodity fund c , $\bar{R}_p$ is the expected return of portfolio p , σ_p is the standard deviation of portfolio p , and ρ_{cp} is the correlation between commodity fund c and portfolio p .

The left side of equation (1) should be familiar to most readers as the Sharpe ratio for evaluating mutual funds. Whether or not a commodity fund enters a portfolio depends on estimates of the expected return and standard deviation of return on commodity funds, the expected return and the standard deviation of the bond-stock portfolio, and the correlation between commodity funds and the portfolio.

We now examine each of these in turn. We will start with correlations and then examine standard deviations. We will save expected return for last since this is the most difficult quantity to estimate.

1. Correlation coefficients. In this section of the paper, we examine the correlation of commodity funds with other investments. Our primary concern is how highly correlated these funds are with stock and bond indexes. However, some interesting insights can be gained into the performance of these funds by examining their correlation with some other indexes.

The arithmetic average of the funds' correlations with seven indexes are presented in table 3. A correlation coefficient for each year is reported along with two average correlation coefficients. The simple average is just the arithmetic average of the six yearly correlation coefficients, one for each year. The overall average is computed by finding the correlation coefficient of each fund with the relevant index using data over the full history of that fund and then averaging across funds.

In order to see if the typical commodity fund should be added to a portfolio of stocks and bonds, we need estimates of the correlation of funds with both stocks and bonds. Note that the correlation with both stocks and bonds is close to zero. The maximum association (coefficient of determination) between commodity funds and stocks is less than 5%. This correlation coefficient is positive in 2 years and negative in 4 years. The simple average of their correlation coefficients is $-.036$, while the overall average is $-.121$.

The correlation coefficients are on average close to and not statistically different from zero. We will use the overall average as our best estimate of the correlation coefficient in later sections. Since this is the smaller value, it gives the maximum chance for commodity funds to enter the portfolio.

When we turn to bonds, we find a parallel case. The largest value for the coefficient of determination is just over 5%. The two estimates of the average correlation are $.007$ and $-.003$. We will use $-.003$, the overall average, in later analysis.

TABLE 3 Correlation Coefficients

Year	Stocks	Bonds	Treasury Bills	Consumer Price Index	Commodity Futures Index	Commodity Cash Index	Average Fund
1	.183	-.021	-.127	.180	.267	.236	.504
2	-.044	-.050	-.065	-.194	-.260	-.114	.541
3	-.003	.228	-.165	-.024	-.402	-.073	.705
4	.050	-.067	-.218	.067	.435	.365	.743
5	-.186	-.203	.194	-.024	.091	.119	.503
6	-.214	.114	.009	-.092	-.543	-.538	.676
Overall average*	-.121	-.003	.010	.009	-.021	-.018	.617
Simple average	-.036	.007	-.062	-.015	-.069	-.001	.612

* The overall average is not an average of the six yearly numbers but rather an average across funds of the correlation of each fund with the respective index for the entire time period over which we have data for the fund.

The other numbers in table 3 are not directly related to the analysis in later sections, but they do shed some light on the characteristics of commodity funds. Commodity funds are often thought of as a hedge against inflation. Note that the correlation with the consumer price index is close to zero. Commodity funds do not exhibit the strong tendency to move with inflation usually attributed to them. Instead they show returns that are almost independent of inflation. The correlation of commodity funds with inflation is less than one-fifth of the correlation of Treasury bills with inflation. As a further test, we ranked funds by correlation with inflation in each of the 6 periods. A Spearman rank correlation was calculated between adjacent periods. While this averaged .10, it was insignificant in each case and sometimes positive and sometimes negative. Thus commodity funds as a group or individually do not serve as a hedge against inflation.

Another interesting point to note from table 3 is that the return on commodity funds is not consistently correlated with the return on either a commodity cash index or a commodity futures index. The two correlations with each index are small, on average, and positive half the time. This indicates that the typical commodity fund does not consistently resemble either of those indexes. The major factor accounting for this probably is the tendency of the funds to sometimes sell and sometimes buy futures contracts, although the tendency of the funds to weight their long positions in futures differently from the indexes is also a contributing factor.

The last column in table 3 shows the average correlation of each fund with an equally weighted index of all funds. This is the only index with which the funds have consistently high correlation. However, if the same analysis were performed for mutual funds, the correlation would be much higher. For example, Sharpe (1966) reports an average correlation of close to .90 between mutual funds and a market index. Thus commodity fund managers do make decisions that lead to similar patterns of returns but not nearly to the extent of managers of common stock mutual funds.

2. *Standard deviation of commodity funds.* Table 1 can be used to estimate the anticipated standard deviation for a randomly selected commodity fund. As noted above, the standard deviation of the typical commodity fund is two to three times the standard deviation of a bond or stock index.

Although there is year-to-year variability in the average standard deviation across funds, it is not so large as to suggest that the future will be much different from the past. What is our best estimate of the standard deviation of the average commodity fund? From the high correlation of returns on individual funds with the fund index, we see that fund returns exhibit strong cross-sectional dependence. If this

dependence were reasonably stable and there were an equal number of funds each year, then an arithmetic average of the standard deviation across all funds and all years would be appropriate.

However, the number of funds varies dramatically over time from 12 in the first year to 85 in the sixth. To use a simple average of the variance on all funds does not take into account this change in the number of funds. This would overemphasize the common element across funds in the sixth year relative to the first year. To adjust for this change in number, we regressed each fund's return against the return on an index of all funds. This divided the total variance into the systematic component and a component unique to each fund for each year. We then calculated an average unique variance by averaging across all funds and all years (266 fund-years). This variance was added to our best estimate of the systematic variance to produce an overall estimate. The details are discussed in Appendix B and the estimate is reported in table 1.

Although the estimate of the average standard deviation of monthly returns from the 6 years of data is likely to be a good estimate of the future standard deviation, it is hard to look at table 1 and feel very comfortable about any estimate of the mean return. It would be nice to have a longer history. Unfortunately, this does not exist. Instead of using table 1 to get an estimate of expected return, we will solve equation (1) for the break-even expected return and then use the data in table 1 to analyze whether or not commodity funds are likely to have returns higher than this break-even point.

3. *Break-even analysis.* The other inputs we need in order to apply equation (1) are the expected return and standard deviation of the optimum bond-stock mix. What values might an investor expect for returns on stocks and bonds and for the covariance structure between them? We used two sets of data for stocks and bonds to estimate these: the last 6 years and the last 25 years of data. Both are shown in table 2. For the 25-year bond index we used an average of long-term government and corporate bonds. This gave us an expected return of .0043 and a standard deviation of .0275. For the 6-year bond index we used the numbers shown in table 2 for the Shearson Bond Index.

If one compares the return on bonds to the Treasury bill rate over the last 6 years and last 25 years, the Treasury bill rate is higher. Unless bond returns are negatively correlated with stock returns (and there is no evidence that they are), bonds will never enter the optimum portfolio. Thus the relevant question is, Should a commodity fund be added to a stock fund? For the 6-year data the inequality is

$$\frac{\bar{R}_c - .0085}{.1130} > \left(\frac{.0131 - .0085}{.0399} \right) (-.121).$$

Solving for $\bar{R}_c$ gives the minimum expected return that a commodity fund must earn to enter the optimum portfolio. This result and a similar calculation for 25 years of data are shown in the first two rows of table 4.

We used a second set of stock and bond return forecasts to obtain break-even returns for commodity funds. The optimal bond-stock split using the full 1926–85 Ibbotson and Sinquefeld data is 63% stock and 37% bonds.¹¹ We took these optimum proportions and applied these proportions to the 6-year and 25-year data shown in table 2. We again solved for the expected return on commodity funds that would just cause commodity funds to enter the optimum portfolio. These results are shown in the last two rows of table 4.

The expected return that would have commodity funds enter is very similar across the four sets of assumptions. It varies between .0069 and .0080 across the four scenarios examined with an average of .0075.

Examining tables 1 and 4 shows that, if a monthly holding period is used, and if we take the 6 years of actual return data as a reasonable estimate of expected returns, then the average return of .0073 is below the average number in table 4 and commodity funds should not enter. If an annual holding period is used and the 6-year average return of $-.0007$ is used as the expected return, the evidence is even stronger that commodity funds should not enter. As discussed earlier, we feel that an annual holding period is the more reasonable assumption. For our sample of 266 fund years, 163 had returns below and 103 had returns above .0075. The average probability of the return on a commodity fund being above .0075 over the 6 years is .42. The odds of the mean return on commodity funds being .0075 or better when the average return over the 6 years is $-.0007$ and the standard deviation of the mean return across the 6 years is .0052 are less than 6%.

An alternative way of examining expected returns for commodity funds is to utilize economic theory to examine what we might logically expect for returns. Commodity managers frequently go short as well as long in the futures contracts. Table 3 shows correlation coefficients of fund returns with futures and spot indexes of $-.021$ and $-.018$, respectively. Only 1% of the return of commodity funds can be explained by index movements. The average correlation is sometimes positive and sometimes negative. These results provide no evidence of a consistent pattern of long or short purchases. If managers were equally likely to be long or short, the expected return before transaction costs, management fees, and management forecasting ability would be zero. This is in contrast to stock and bond funds that have substantial positive

11. This was obtained by solving standard first-order conditions. The calculations are available from the authors.

TABLE 4 Break-even Rates of Return

Scenario	% Stock	Excess Return ($\bar{R}_p - R_F$)*	SD (σ)*	Correlation with Portfolio ρ (cp)†‡	Break-even Rate of Return ($\bar{R}_c$)‡
6 year	100	.0046	.0399	-.121	.0069
25 year	100	.0023	.0414	-.121	.0077
6 year	63	.0033	.0309	-.099	.0073
25 year§	63	.0012	.0289	-.110	.0080

* Entries from table 2.

† The covariance of a commodity fund with a portfolio is the sum of the proportion in each asset in the portfolio times the correlation of the commodity fund with the asset times the product of the standard deviation of commodity funds times the standard deviation of the asset. For example, the covariance for the third entry is $.63(-.121)(.1130)(.0399) + .37(-.003)(.1130)(.0293)$.

‡ Assumes standard deviation of commodity funds of .1130 (see table 1).

§ Assumes a correlation of .10 between stocks and bonds, which is the average correlation between corporate and government bonds and the stock index, as shown in Ibbotson (1985).

expected returns before transaction costs and management fees, even with no management forecasting ability.

Transaction costs should not be taken lightly. Even though costs per transaction are low, transactions are frequent. Irwin and Brorsen (1985) find commissions averaging 10.7% for the 20 firms they sample. In addition, the management fee on commodity funds is substantially higher than the management fee on stock or bond funds. There are two types of management fees on commodity funds: a fee related to asset size and a fee related to performance. For the funds in our sample, asset fees ranged 3%–6% per year. This is in contrast to stock and bond funds where one-half of 1% is usual. In addition, there is an incentive fee. Examining table 1 might suggest that incentive fees are unimportant. However, given the tremendous variability of monthly returns and the basing of incentive fees on shorter-term performance, incentive fees can be high even with poor long-term performance. Irwin and Brorsen (1985) find that management and incentive fees average 8.5%. Thus total yearly fees of 19.2% can be expected. In contrast, Sharpe (1981) reports that total fees for mutual funds average about 1% per year.

Examining table 4 shows that commodity managers must earn a return of about 9% per year to be attractive investments. Adding this to transaction costs and management fees of above 19% and an expected return that is close to zero without forecasting ability suggests that a return to forecasting ability of over 28% is necessary for commodity funds to be attractive. Evidence from other security markets should lead to a healthy degree of skepticism.

Thus actual returns of commodity funds as well as the more general evidence of financial economics suggest that a randomly selected com-

modity fund should not be held. In the next section we will examine whether there are characteristics of commodity funds that allow funds that will have above-average performance to be selected.

III. Prediction of Superior Performing Funds

Ex post, there is always a commodity fund that did best. Given the high variance in fund performance (both over time for any fund and cross sectionally among funds), it is not at all surprising that at any point of time or over any short period of time there always exists a fund that did well. The relevant question for an investor is, Can he or she identify, ex ante, the funds that will do well? In this section, we shall examine whether any of several attributes of fund performance is predictive of future performance. The degree of consistency and predictability of performance is interesting in its own right. It also has major implications for the inclusion of commodity funds in optimal portfolios.

This section is divided into two parts. In the first part, we discuss the tests of consistency and predictability we use as well as the performance series we look at. In the second part, we discuss empirical results.

A. Consistency and Predictability

The majority of our analysis in this section is based on two types of tests. The first type is a test of consistency. That is, do funds that have high values on a measure in one period tend to have high values in the subsequent period? Two separate tests of consistency will be used. The first measures consistency across all funds, while the second examines consistency only for those funds that have done extremely well or extremely poorly. The second type is a test of predictability, namely, whether past values of a performance measure forecast better than a naive model.

1. *Consistency.* To test consistency we examined performance both across all funds and for the tails. In order to judge whether overall performance in one year was associated with performance in the next year, Spearman's rank correlation was computed between all adjacent years for all funds that were present in both of the paired years.¹² These results are contained in table 5.

For example, all funds that existed in the first year of our sample period were ranked by return from high to low in that year. The return of each of these funds was computed for the second year of our sam-

12. Correlation coefficients were also computed on the unranked (raw) data. The results were so similar to the rank correlations that we have not bothered to report them or to discuss them further in this paper.

TABLE 5 Spearman Rank Correlations

	10 Observations: Year 1 vs. Year 2		15 Observations: Year 2 vs. Year 3		34 Observations: Year 3 vs. Year 4		49 Observations: Year 4 vs. Year 5		67 Observations: Year 5 vs. Year 6	
	Correlation	t-Value ^a	Correlation	t-Value ^a	Correlation	t-Value ^b	Correlation	t-Value ^b	Correlation	t-Value ^b
Sharpe ratio: Year t vs. Year $t + 1$ ^c	.2242	...	.4393	**	.0591	.335	-.1008	-.695	.2886	2.43*
Returns: Year t vs. year $t + 1$	.0667	...	.4679	**	.1499	.858*	.1023	.705	.3068	2.600*
SD: Year t vs. year $t + 1$	.9152	*	.4464	**	.1704	.978	.6543	5.932*	.3357	2.873*
SD: Year t vs. Sharpe ratio year $t + 1$	-.6485	**	-.4429	**	-.4460	-2.819*	-.0548	-.376	-.1923	-1.580

^aFor small samples (under 30), the statistical significance of any correlation was arrived at using small sample tests based on the number of permutations possible. For a discussion of the tests, see Siegel (1956, pp. 210, 211); and for critical values, see Siegel (1956, table P, p. 284).

^bFor large samples (over 30), t -values were computed using Kendall's procedure. See Siegel (1956, p. 212) for a discussion of the procedure.

^cThese tests are conducted in terms of monthly returns assuming an annual holding period. The tests were also run on monthly returns assuming a monthly holding period. The results are very similar, and, in the interest of brevity, the second set of results have not been reported.

*Statistically significant at the 1% level.

**Statistically significant at the 5% level.

ple, and funds were again ranked from best to worst. The Spearman correlations were computed between the ranks for the first and second years. In the case of the first and second set of paired years, small sample tests of Spearman's coefficient were used.¹³ For the third, fourth, and fifth paired sample years, Kendall's large sample test was used to compute the adjusted correlation coefficient, which is distributed as Student's t .¹⁴

A second set of tests were performed to examine consistency. That is, if we selected the funds that had unusually high (or low) values on a performance characteristic in one period, would we get unusually high or low values for that or another important characteristic in the next period? For example, the return from purchasing the three funds that had the highest return and holding them for 1 year was compared with the return from purchasing and holding the three funds with the lowest return. Similarly, the return from purchasing and holding the top third, middle third, and bottom third of funds in each period was computed and compared. Even if there is no consistency across the population of funds from period to period, it is possible that there is consistency in the very best and worst performing funds.

2. Predictability. While a measure of association is interesting in itself, perhaps the more important set of tests for our purposes is whether past values of a performance measure can be used to predict future performance. To test this, we examine whether past values of a performance measure are better predictors than the most naive possible prediction (see tables 6–9). In all cases (except for one noted in the next section), the naive prediction used is that the best estimate for all funds is zero. For example, in the case of predicting returns, we compare the prediction that next period's return equals this period's return with the prediction that next period's return equals zero. If past return is a better predictor than zero, then past return allows one to differentiate between funds.

For each fund present in a particular year, we predict performance in the subsequent year. We examined five pairs of adjacent years. For each fund, we computed the squared forecast error under each of our two forecasts. Since we have related samples (two forecasts for each fund), the properties of the mean difference in squared forecast error were examined. The average difference in squared forecast error and the standard deviation of the mean difference in squared forecast error was computed and the associated t -value of the difference is reported in table 10.

A positive value in this table indicates that the naive model led to

13. See Siegel (1956, pp. 211, 212, and table F).

14. See Siegel (1956, pp. 212, 213).

TABLE 6 Values of Sharpe at $t + 1$ from Ranking by Sharpe Ratio in t

	Period 2		Period 3		Period 4		Period 5		Period 6		Avg. Ratio	Avg. Rank
	Sharpe Ratio	Rank	Sharpe Ratio	Rank	Sharpe Ratio	Rank	Sharpe Ratio	Rank	Sharpe Ratio	Rank		
Top 3	.3279	1	.1031	1	-.1834	2	-.0933	2	.3392	1	.0987	1.4
Bottom 3	.2898	2	-.1966	2	-.1032	1	.1262	1	.0770	2	.0386	1.6
Top 1/3*	.3279	1	.0569	1	-.2427	1	-.1903	2	.0622	1	.0028	1.2
Middle 1/3	-.1154	3	-.0567	2	-.4090	3	-.2308	3	-.0488	2	-.1495	2.6
Bottom 1/3	.2898	2	-.1685	3	-.2691	2	-.1115	1	-.1613	3	-.0841	2.2

* In forming the three groups, if the total number of firms was not divisible by 3, the extra one or two firms were placed in the middle group.

TABLE 7 Value of Return at $t + 1$ for Ranking of Return at t

	Period 2		Period 3		Period 4		Period 5		Period 6		Avg. Return	Avg. Rank
	Return	Rank	Return	Rank	Return	Rank	Return	Rank	Return	Rank		
Top 3	.0167	1	.0170	1	-.0229	1	-.0109	2	-.0044	2	-.0009	1.4
Bottom 3	.0149	2	-.0054	2	-.0285	2	.0249	1	.0136	1	.0039	1.6
Top 1/3*	.0167	1	.0127	1	-.0162	1	-.0009	2	.0152	1	.0059	1.2
Middle 1/3	.0050	3	.0118	2	-.0351	3	.0035	1	.0003	3	-.0029	2.4
Bottom 1/3	.0149	2	.0045	3	-.0275	2	-.0127	3	.0018	2	-.0038	2.4

* In forming the three groups, if the total number of firms was not divisible by 3, the extra one or two firms were placed in the middle group.

TABLE 8 Value of Standard Deviation at $t + 1$ for Ranking of Standard Deviation at t

	Period 2		Period 3		Period 4		Period 5		Period 6		Avg. SD	Avg. Rank
	SD	Rank	SD	Rank	SD	Rank	SD	Rank	SD	Rank		
Top 3	.2300	1	.1131	1	.1795	2	.1599	1	.0364	2	.1438	1.4
Bottom 3	.0637	2	.0639	2	.1955	1	.0346	2	.0639	1	.0843	1.6
Top 1/3*	.2300	1	.0934	1	.1244	1	.1077	1	.1326	1	.1376	1.0
Middle 1/3	.1183	2	.0749	2	.1074	3	.0880	2	.0915	2	.0960	2.2
Bottom 1/3	.0637	3	.0648	3	.1127	2	.0525	3	.0678	3	.0723	2.8

*In forming the three groups, if the total number of firms was not divisible by 3, the extra one or two firms were placed in the middle group.

TABLE 9 Values of Sharpe Ratio at $t + 1$ for Ranking of Standard Deviation at t

	Period 2		Period 3		Period 4		Period 5		Period 6		Avg. Ratio	Avg. Rank
	Sharpe Ratio	Rank	Sharpe Ratio	Rank	Sharpe Ratio	Rank	Sharpe Ratio	Rank	Sharpe Ratio	Rank		
Top 3	-.1312	2	.0520	2	-.3518	2	-.0765	1	-.5559	2	-.2127	1.8
Bottom 3	.5380	1	.1335	1	-.1032	1	-.3483	2	.1481	1	.0736	1.2
Top $\frac{1}{3}$ *	-.1312	3	-.0992	3	-.3722	3	-.2768	3	-.1229	2	-.2005	3.0
Middle $\frac{1}{3}$	.0420	2	-.0697	2	-.3637	2	-.0552	1	.0077	3	-.0878	1.6
Bottom $\frac{1}{3}$	.5380	1	.1160	1	-.1890	1	-.2116	2	-.0353	1	.0436	1.4

*In forming the three groups, if the total number of firms was not divisible by 3, the extra one or two firms were placed in the middle group.

TABLE 10 t -Values of Squared Error of Forecast Model Minus Naive Forecast

	10 Observations: Years 1 and 2	15 Observations: Years 2 and 3	34 Observations: Years 3 and 4	49 Observations: Years 4 and 5	67 Observations: Years 5 and 6
Sharpe ratio at t as forecaster of Sharpe ratio at $t + 1^*$	-.05	1.48	-1.72	.81	2.18
Returns at t as forecaster of return at $t + 1^*$	.49	2.04	4.94	2.36	2.57
SD at t as forecaster at SD at $t + 1^\dagger$	-1.03	.89	1.02	-.60	.25

*These tests were conducted in terms of monthly returns assuming a yearly holding period. Similar results were obtained when monthly returns assuming a monthly holding period were used. The naive model was zero for all funds.

†The naive model was the average (across all funds) SD for the previous year. The values used were .1577, .1211, .0824, .1167, and .0793, respectively.

lower forecast errors. For the later three pairs of years, the samples are large enough that, by the central limit theorem, the statistic should be normally distributed. A one-tailed test is used to see if past performance predicts better than the naive model. Statistical significance is indicated in table 10. For the first two pairs of years, the sample is too small to resort to the central limit theorem. Here, the Wilcoxon Matched Pairs Signed-Ranks Test was used to judge statistical significance.¹⁵ In table 10, *t*-values are reported to show how many standard deviations away from zero the mean is, even though they are not used to test significance.

B. The Empirical Results

Examining equation (1) shows that the condition for commodity funds to enter into an optimal portfolio is the Sharpe performance index. In this section, we first examine the Sharpe ratio itself to see if its value across funds is consistent and predictable over time. We then examine each of its components, return and standard deviation, in turn. If either of these shows predictability, we will then examine whether it can be used to better differentiate between funds with respect to their Sharpe ratios. In all cases we will use monthly returns based on an annual holding period. Results for a monthly holding period were very similar.

1. The Sharpe ratio. The first question examined was, Do funds that have a high value for the Sharpe ratio in one period tend to have a high value for the Sharpe ratio in the subsequent period?

From table 5, the average of the five rank correlations for the Sharpe index for adjacent years is 0.182. While it is statistically different from zero at the 1% level in 1 out of 5 years, it is negative in 1 year and the average association is less than 4%.

When we examine the tails of the distribution (table 6), we find a slight amount of information in the Sharpe coefficients. On average, the top three funds (and the top third of funds) ranked by the Sharpe ratio have higher values for their Sharpe ratio than the bottom three funds (or bottom third of funds). Lest we place too much emphasis on these results, we should remark that, while the top three funds do better than the bottom three on average, they do worse in 2 of the 5 years. Furthermore, while the top third of funds does better than the bottom third on average, the bottom third does better than the middle third on average. Based on these results, it appears unlikely that using the Sharpe ratio to select funds will lead to improvement.

When we examine (table 10) whether this period's Sharpe ratio is a better predictor of next period's ratio than is zero, we get ambiguous

15. See Siegel (1956, pp. 75-83, and table G).

results. Past Sharpe ratios forecast better in 1 year and worse in 4 years. They never forecast better at a statistically significant level, while the naive model performs best in 1 year at a statistically significant level.

We now turn to the components of the Sharpe ratio, return and standard deviation, to see if they show consistency and predictability over time.

2. Returns. From table 5 we see that the average of the five rank correlation coefficients in monthly returns assuming an annual holding period is 0.219. The correlation is positive in each sample case and is significantly different from zero at the 5% level in two of the five samples. While the association is always positive, the extent of association (less than 5%) is relatively small.

Table 7 shows that, if an investor bought the three funds that performed the worst and held them for 1 year, on average the investor would do very slightly *better* than buying the three funds that performed best. One would have done better in 2 years and worse in 3 years. If the sample is divided into three groups according to returns, the best performance, on average, would have been obtained by buying the third of the funds that performed best. The consistency of the return measure appears to be marginally better than the consistency of the Sharpe index. However, the change in ranking according to how the tails are defined warns us not to place undue reliance on this criterion. When we examine predictability of past returns (table 10), we find that the naive forecast of zero return for each fund is a better forecast of next year's average monthly return than assuming that next year's return equals last year's return. In fact, if a two-tailed test were used, the naive model would produce lower error at the 5% significant level in four out of five cases. Clearly, past returns of any fund provide no information for predicting future returns.

3. Risk. While past return might not tell the potential investor anything about the kind of returns he or she should expect to receive in the future, the past would still be useful if it could allow the investor to differentiate between funds in terms of riskiness. The measure of riskiness we examined was the standard deviation of fund returns.¹⁶

There does seem to be consistency in standard deviation. The average rank correlation (across the five sets of paired years) between standard deviation in one period and standard deviation in the next (table 5) was .0514. This is the highest value for any of our measures of

16. In addition, the rank correlations and naive predictions were examined for betas and correlations of the funds with the common stock index, the bond index, and an index of commodity bond performance. No association or predictive power was found.

performance. Furthermore, the average rank correlation across funds in each of the paired years is always positive, and it is statistically significantly different from zero on average and in 4 of the 5 paired years.

Turning now to an examination of the tails (table 8) of the distribution, we see evidence that past ordering by risk provides information about future ordering by risk. When we examine the top three and bottom three funds, the top three have the highest risk on average and in 3 out of 5 years. When we look at the three equally sized groups, the top third of the funds has the biggest risk on average and in each of the 5 years. There seems to be information about the future risk contained in past risk for firms with very high or low standard deviations.

In examining prediction for the other variables, we have used zero as the naive forecasting model. However, this seems inappropriate (too naive) in the case of standard deviation, since we know that all values have to be greater than or equal to zero. Instead of zero, we used as the naive forecast of next year's standard deviation for each fund the average standard deviation (across funds) for the prior year. Thus the naive model forecasts all standard deviations as identical and the naive model should perform better if differences in the standard deviation of funds from the mean in the past contain no information about future differences. The results do not allow us to conclude that past standard deviations are a useful predictive tool. However, past standard deviations do outperform the naive model in 3 of our 5 paired years. While past risk does not seem to be a good predictor of future risk, there is fairly strong evidence that past values of risk can be used to identify those funds that will be most (or least) risky in future periods.

4. Risk as a predictor of desirability. We have seen that, while there is at best weak evidence of return or the Sharpe ratio being able to rank funds according to future values of these measures, there is reasonably strong evidence of persistence in the relative riskiness of funds. The purpose of this section is to see whether selecting funds on the basis of standard deviation leads to improvements in the Sharpe ratio.

Table 5 shows the rank correlation of the standard deviation of funds in each year with the Sharpe ratio for funds in the next year. The relationship is negative in 5 of 5 paired years. It is statistically significantly different from zero (and negative) in 3 years. Thus, low standard deviation funds are associated with higher Sharpe ratios. The average rank correlation is $-.357$, which is considerably larger in absolute magnitude than the rank correlation between the Sharpe ratio in successive periods.

In table 9, we present the results (in terms of the Sharpe ratio obtained 1 year later) of purchasing the top three funds, the bottom three funds, and the top third, middle third, and bottom third of funds ranked

by their standard deviation. On average and in 4 out of 5 years, the best Sharpe ratio is obtained by purchasing the funds with the lowest standard deviation. There does appear to be information present in these rankings.

Both the association of standard deviation in one period with standard deviation in the next and the association of standard deviation and the Sharpe ratio in the next period suggest that standard deviation might be useful in picking good commodity funds. Examining table 8 suggests that standard deviations of .07-.14 might be possible by selecting a low or high standard deviation fund in the prior period. Substituting this range into equation (1) gives the break-even return necessary to include a commodity fund ranging from .0067 to .0075 for the 6-year 100% stock case with similar results for the other scenarios. Thus the predictability of standard deviations results in almost no change in the break-even rate.

Furthermore, it is ambiguous whether a high or low standard deviation fund is desirable. Examining equation (1) shows that, for returns below the risk-free rate, a high standard deviation is desirable.

An investor can probably select a fund with a standard deviation different from the average. Furthermore, low standard deviations have some association with higher Sharpe ratios. However, given the minimal impact on the break-even rate and the ambiguity whether small or large standard deviations are desirable, the predictability of standard deviation is unlikely to affect the basic decision whether commodity funds should be selected in the first place.

IV. Conclusion

This paper is the first comprehensive review of publicly traded commodity funds.¹⁷ The returns on these funds are highly variable, with a standard deviation two to three times that of bonds or common equity indexes. This variability was fairly consistent from year to year, and we expect that estimates from our 6-year period will be sustained in the future. The correlation structure is also fairly stable from year to year. Calculating a break-even expected return and comparing it with actual returns make it doubtful that public commodity funds should be included in an investor's portfolio.

When we examined whether we could select a superior commodity fund on the basis of past performance the answer was probably not. Using the past value of the Sharpe ratio as a predictor led to almost no improvement in performance. This was true whether we used the full set of funds or just the extremes. A similar result was obtained with

17. Lintner (1983) discussed results for eight publicly traded funds that existed over a 3½-year period.

returns. However, there is predictability in standard deviation. A fund with a lower or higher standard deviation than the average can probably be selected on the basis of past values. However, this has minimal impact on the break-even rate and hence has little impact on the desirability of commodity funds.

Appendix A

The condition for a security to enter an optimal portfolio is easy to derive using the simple rules of Elton, Gruber, and Padberg (1976). Consider an asset and a portfolio. Since there is a single portfolio and a single asset, there is a single correlation coefficient of interest. Thus the assumption of a constant correlation coefficient can be made without loss in generality. As Elton et al. (1976) have shown, a security will enter in a positive amount if

$$\frac{\bar{R}_i - R_F}{\sigma_i} - \frac{\rho}{1 - \rho + N_k \rho} \sum_{j \in k} \frac{\bar{R}_j - R_F}{\sigma_j} > 0,$$

where $\bar{R}_i$ is the expected return on asset i , R_F is the riskless rate of interest, ρ is the correlation coefficient, N_k is the number of securities, and k is the set of included securities. With a single asset and a single portfolio, N_k and K are both one and the condition becomes

$$\frac{\bar{R}_i - R_F}{\sigma_i} - \rho \left(\frac{\bar{R}_p - R_F}{\sigma_p} \right) > 0.$$

This is equation (1) above. The same condition can be derived in a more cumbersome fashion using first-order conditions.

Appendix B

To estimate the variance, we ran the following single-index model for each year y :

$$R_{ity} = \alpha_{iy} + \beta_{iy} I_{yt} + \epsilon_{ity},$$

where R_{ity} is the return in month t of year y for commodity fund i ; I_{yt} is the value of the index in month t of year y , in which the index is an equally weighted index of all commodity funds; β_{iy} is the responsiveness of commodity fund i to changes in the index in year y ; ϵ_{ity} is a random variable for fund i in month t of year y and represents unsystematic return; and α_{iy} is a constant in year y for fund i .

The total variance of fund i in year y is given by

$$\sigma_{iy}^2 = \beta_{iy}^2 \sigma_{Iy}^2 + \sigma_{\epsilon iy}^2,$$

where σ_{Iy}^2 is the variance of the index in year y and $\sigma_{\epsilon iy}^2$ is the residual variance for fund i in year y .

Since the residuals are independent over time and the removal of the average fund performance makes them very close to cross-sectionally independent, the

best estimate of the average variance of the residuals is

$$\sigma_e^2 = \frac{\sum_y \sum_i \sigma_{eiy}^2}{266}.$$

Similarly, the best estimate of the market index variance is an average across the 6 years:

$$\sigma_I^2 = \frac{\sum_y \sigma_{Iy}^2}{6}.$$

Our best estimate of the yearly variance for an average fund is the sum of the average beta squared times the variance plus the average unique variance:

$$\bar{\beta}_I^2 \sigma_I^2 + \sigma_e^2.$$

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